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Characteristics of Dynamical Phase Transitions for Noise Intensities

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Abstract

We simulate and analyze dynamical phase transitions in a Boolean neural network with initial random connections. Since we treat a stochastic evolution by using a noise intensity, we show from our condition that there exists a critical value for the noise intensity. The nature of the phase transition are found numerically and analytically in two connections of probability density function and one random network.

1 Introduction

Boolean neural networks have been described as generic models for the dynamics of complex systems of interacting entities, such as social and economic networks, neural networks, and gene or protein interaction networks [1]. Kauffman [2] as a model for gene regulation was introduced and studied the simplest and most widely neural network models. Derrida and Pomeau [3] have performed calculations of random automata model by using a Boolean function. Their work has given annealed approximations and quantitative predictions for distances between iterated configurations.

Boolean networks have been used to describe various models in complex systems such as neural networks with associative memory [4-5], spin glasses [6-9], dynamics of evolution [10-11], and cellular automata [12-13]. It is well known that a typical Boolean network consists of a set of binary elements which are connected among them to indicate a net, and the use of common tools has revealed a robust parallel between Boolean networks and dynamical systems. Historically, neural networks have approximated universal and nonlinear functions with arbitrary accuracy [14]. This approximate method is an important advance for neural networks, because of the huge number of possible nonlinear patterns for real world problems. Neural networks have been described to be

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effectively in modelling and forecasting nonlinear time series with noise [15]. Until now, many scientists [16] have made the comparison between the neural network and the traditional method in time series modelling and forecasting performances.

Furthermore, over the last two decades, the remarkable potential of complex networks to simulate and analyze the dynamical behavior of complex systems has gradually been an increasing trend in new fields of research in the social, natural, engineering, and medical sciences. In the network theory, the small-world and scale-free network models [17,18] have been studied widely in various applications of the scientific fields. The two network models have played a crucial role in the complex phenomena [19-21], and of current interest are scale-free networks because it follows the power law for its degree distribution. Particularly, we now introduce the random neural network to our paper, and the problem of scale-free neural networks will be appeared in other paper.

Until now, several papers have analyzed the non-equilibrium dynamics of deterministic Boolean neural networks [22,23] and suggested the existence of a variety of possible collective behaviors such as synchronized oscillations or chaos [24,25]. The influence of noise on the dynamics of Boolean networks has been analyzed in several published papers [26,27] as well. Scientists researched on neural networks have been interested in considering the changes in the dynamical properties of a deterministic system in the presence of noise. Following this motivation, we study random network models exhibiting self-organization and analyze its tolerance to the effect of noise. In this paper, we mainly show from two connections (of probability density function) and one random network that the system undergoes a dynamical phase transition as its amount of randomness is increased.

2 Theoretical Background

Consider a neural network composed of *N* elements, each of which can only take the values $\sigma_i = +1$ or $\sigma_i = -1$. Every σ_i is randomly connected to any *L* elements of the network, which define its set of linkages. The parameter *L* is the connectivity of the network, and each linkage is weighted by an independent random variable. The *NL* connections of a network and its corresponding weights remain fixed throughout the evolution of the system. In our model, the input function [28] at discrete time step *t* is represented in terms of

$$I(c_{i_1}, c_{i_2}, ..., c_{i_L}; \sigma_{i_1}(t), \sigma_{i_2}(t), ..., \sigma_{i_L}(t)) = Sign\{\sum_{j=1}^{L} c_{ij}\sigma_{i_j}(t)\}. \tag{1}$$

Here $\sigma_i(t)$ is connected to any *L* elements having its set of linkages $\{\sigma_{i_j}(t)\}$ for *j* = 1,2,...,*L*, and each linkage $\sigma_{i_j}(t)$ is weighted by an independent random variable c_{ij}. The input function takes the same value as the majority of the linkages, if it corresponds to the majority rule.

Using Eq. (1), we introduce a stochastic evolution rule for $\sigma_i(t+1)$ with a noise intensity γ such that

$$\sigma_i(t+1) = I(c_{i_1}, c_{i_2}, ..., c_{i_L}; \sigma_{i_1}(t), \sigma_{i_2}(t), ..., \sigma_{i_k}(t)) \tag{2}$$

with $1-\gamma$ and

$$\sigma_i(t+1) = -I(c_{i_1}, c_{i_2}, ..., c_{i_L}; \sigma_{i_1}(t), \sigma_{i_2}(t), ..., \sigma_{i_k}(t)) \tag{3}$$

with γ. In above equation, we can select randomly a varying noise intensity γ between 0 and 1/2. In the case with $\gamma = 0$, a neural network system at time $t+1$ will converge to an ordered state in which all the $\sigma_i(t+1)$ are equal.

Next, we consider that the neural network system undergoes a dynamical phase transition from an ordered to a disordered state as the noise intensity is increased. In order to define the order parameter adequately described the degree of alignment of the elements of the network, we introduce a statistical quantity as

$$\sigma(t) = \frac{1}{N}\sum_{i=1}^{N}\sigma_i(t), \tag{4}$$

where $|\sigma(t)| \to 1$ for an ordered system in which all elements take the same value, while $|\sigma(t)| \to 0$ for a disordered system. For systems where the time-average of $|\sigma(t)|$ converges, an order parameter Φ is defined as

$$\Phi = \frac{1}{t-t_o}\sum_{t=t_0}^{t}|\sigma(t)|, \tag{5}$$

where t_0 can take any arbitrary finite time without changing Φ . From Eq. (5), it is well known [29] that the phase transition is described by

$$\Phi = [C(\gamma_C - \gamma)]^{1/2} \tag{6}$$

for $\gamma_C - \gamma = 1$, and

$$\Phi = 0 \tag{7}$$

for $\gamma_C < \gamma$.

3 Numerical Calculations and Results

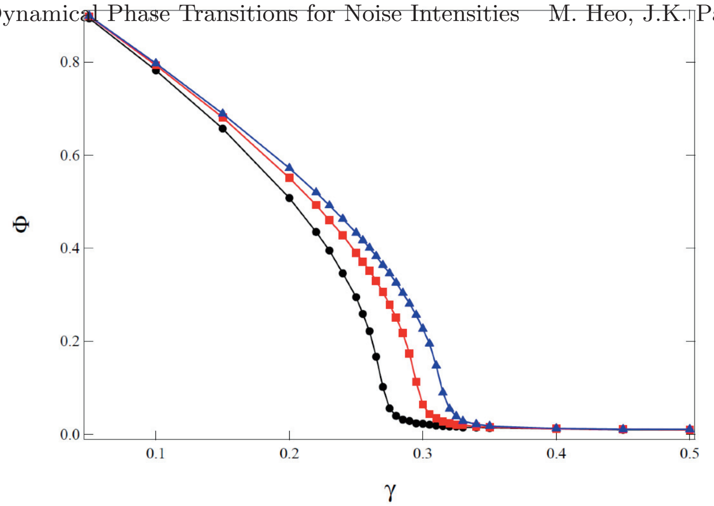

Figure 1: Values of order parameter Φ for a number of links L =7 (circle), 9 (square), and 11 (triangle) in the random network.

In this paper, the N elements of our neural network can be extended to a larger number, but we only restrict ourselves to the cases for which the computer simulations are carried out for $N = 5\times10^3$ elements. From random initial conditions, we simulate numerically the evolution of the model for a neural network with $N = 5\times10^3$ elements and three kinds of connectivities $L = 7,9,11$. Next, the order parameter Φ is obtained by integrating $|\sigma(t)|$ from $t_0 = 1\times10^3$ until $t = 1\times10^4$. We treat three cases as two connections (of probability density function) and one random network.

We can calculate the results for the case of a network with fixed connection weights c_{ij} following that the probability density function $P(x)$ is equal to 1 if $0 \le x \le 1$ and 0 otherwise. The phase transitions occur only at $\gamma_C = 0.241$, 0.264 and 0.285 for L =7, 9, and 11, respectively. The numerical results find the bifurcation diagram of Φ as a function of the noise intensity for the case with $c_{ij} = 1$, in which all connection weights are equal. As the input function then becomes the majority rule, the system undergoes apparently a phase transition with $\gamma_C = 0.283$, 0.299 and 0.325 for L =7, 9, and 11. For $\gamma < \gamma_C$, all elements in the system will tend to align either to +1 or to −1. Figure 1 shows Φ as a function of the noise intensity for the case with $c_{ij} = 1$ in the random network with a number of nodes $N = 5\times10^3$ and links L =7, 9, 11. The system undergoes a phase transition with $\gamma_C = 0.269$ (L =7), 0.297 (L=9), and 0.313 (L=11), and the computer-simulation is averaged 100 ensembles in Fig. 1.

4 Codnclusions

We have simulated and analyzed dynamical phase transitions in a Boolean neural network with initial random connections. We have ascertained the nature of phase transitions numerically and analytically in two connections and one random network. Due to the randomness and the noise with initial linkages in the neural network, the statistical properties in the dynamics may not change if the connection weights or the linkages are either time-independent or if they are randomly re-assigned at every time step. Through Boolean networks, it means that the annealed and quenched dynamics are equivalent for our model presented in this paper. In other cases, random energy models used by

Derrida [30,31] have extended to study spin glasses and the protein folding problems [32]. The connection between Random energy models and traditional methods of statistical mechanics has been discussed by Janzen *et al.* [33] in the context of Levy spin glasses. Until present, there exist a number of reasons to use the neural network for time series simulation and analysis. If an function has relation between the inputs and outputs for any forecasting model, then it is very important to identify accurately this function. All these features have made neural networks useful for time series modelling and forecasting [34,35] in real world problems. In the future, we will extend our model to scale-free networks of other scientific fields.

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